Effective fsky for general weights and depth map

$$\mathrm{Cov}(C_\ell) = \frac{2}{2\ell+1}\left[\frac{1}{f_{sky}^{sig}} C_\ell^2 + \frac{2}{\hat{f}_{sky}^{cross}} C_\ell N_\ell + \frac{1}{\hat{f}_{sky}^{noise}} N_\ell^2\right]$$

with

$$f_{sky}^{signal} = f_{sky} \frac{\langle w_i^2 \rangle^2}{\langle w_i^4 \rangle}$$

average in non-zero pixels

$$\hat{f}_{sky}^{cross} = f_{sky} \frac{\langle w_i^2 \rangle \langle w_i^2 \sigma_i^2 \Omega_{pix} \rangle}{\langle w_i^4 \sigma_i^2 \Omega_{pix} \rangle}$$

$$\hat{f}_{sky}^{noise} = f_{sky} \frac{\langle w_i^2 \sigma_i^2 \Omega_{pix} \rangle^2}{\langle w_i^4 (\sigma_i^2 \Omega_{pix})^2 \rangle}$$

To find the rescaling of the covariance matrix, we also need

$$N_\ell = \underbrace{\frac{\langle w_i^2 \sigma_i^2 \Omega_{pix} \rangle \left\langle \frac{1}{\sigma_i^2 \Omega_{pix}} \right\rangle f_{sky}}{\langle w_i^2 \rangle}}_{f^{noise}_{eff}} \underbrace{\frac{\left\langle \frac{1}{(\sigma_i^2 \Omega_{pix})^2} \right\rangle}{f_{sky} \left\langle \frac{1}{\sigma_i^2 \Omega_{pix}} \right\rangle^2}}_{\text{previous } f_{sky}^{noise}} \underbrace{N_\ell^{inv}}_{\text{Noise power for inverse noise variance weighting}}$$

So the signal rescales as before.

$$\frac{1}{\hat{f}_{sky}^{noise}} N_\ell^2 = \frac{1}{\hat{f}_{sky}^{noise}} \frac{\left(f_{eff}^{noise}\right)^2}{f_{sky}^{noise}} \frac{1}{f_{sky}^{noise}} \left(N_\ell^{inv}\right)^2$$

$$= \underbrace{\frac{\left(f_{eff}^{noise}\right)^2}{\hat{f}_{sky}^{noise} f_{sky}^{noise}}}_{\text{rescaling}} \frac{1}{f_{sky}^{noise}} \left(N_\ell^{inv}\right)^2$$

Of course if a fraction of effort is used it enters as $\frac{1}{r^2}$ as before.

$$\frac{1}{\hat{f}_{sky}^{cross}} N_\ell = \frac{1}{\hat{f}_{sky}^{cross}} \frac{f_{eff}^{noise}}{f_{sky}^{noise}} N_\ell^{inv}$$

$$= \underbrace{\frac{f_{sky}^{cross}}{\hat{f}_{sky}^{cross}} \frac{f_{eff}^{noise}}{f_{sky}^{noise}}}_{\text{rescaling}} \frac{1}{f_{sky}^{cross}} N_\ell^{inv}$$

For fraction of effort r, there is an additional $\frac{1}{r}$